Kuwait University | **MATH 101** | **Date: July 5, 2007**
Dept. of Math. & Comp. Sci. | **First Exam** | **Duration: 75 mins.**

1. Use the (ε, δ) definition of the limit to show that:

$$\lim_{x \to 1} \left(3 - \frac{1}{2}x\right) = \frac{5}{2}.$$

(3 pts)

2. Evaluate the following limits (if they exist):

a) $\lim\limits_{x \to 2} |x - 2| \cos^2\left(\dfrac{1}{x-2}\right)$ (3 pts)

b) $\lim\limits_{x \to 3} \left(\dfrac{x^3 - 27}{\sqrt{x} - \sqrt{3}}\right)$ (3 pts)

3. Find the equation of the normal line to the curve of

$$y = x^2 + \sec x \tan x$$

at $x = 0$. (3 pts)

4. Find the vertical and horizontal asymptotes (if any) for the function

$$f(x) = \frac{|x - 1|\,(2x + 1)}{2x^2 + x}.$$

(4 pts)

5. a) State the Intermediate Value Theorem. (1 pt)

b) Let $f(x) = x^3 - 1$ and $g(x) = -x^4 + 2x$. Use the Intermediate Value Theorem to show that the graphs of f and g intersect at least once.
(3 pts)

6.

$$\text{Let } f(x) = \begin{cases} A\left(\dfrac{\sqrt{2x-1} - \sqrt{3x-2}}{x^2 + 3x - 4}\right) & if\ x < 1, \\[2ex] -\dfrac{1}{10} & if\ x = 1, \\[2ex] \dfrac{B\,|x - 1|}{(x^3 + 1)\sin(x - 1)} & if\ x > 1. \end{cases}$$

a) Find the values of A and B so that $f(x)$ is continuous at $x = 1$.
(3 pts)

b) Find the relation between A and B so that $f(x)$ has a jump discontinuity at $x = 1$.
(2 pts)

Good luck